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Notes for a mathematical *aisthesis*: from *Gestalt* to language

> To *comprehend* something means to have it as a gestalt, or at its functional place in a gestalt.
>
> (Duncker 1926, p. 653)

I would like to begin with a short story that has to do with the renovation of a house. Here, where I currently live and where I am now writing, it had been decided to renovate the floors; at the time it was my turn to accompany Antonio (I use a fictitious name) on the first sweep of the area. Mr. Antonio, just under 70 years old, the father of the renovation company manager, had had to leave school when he was very young; he was retired by then, but loved to make himself useful with his son's clients. I had the land registry file with me and was about to give him the information about the surface area, when, with a smile, he motioned me to wait and pulled out from his coat a squared notebook, a pencil and a tape measure. He started a careful observation of the ground, took a few steps into the still bare open space with its highly irregular floor plan, took two lengths as if to establish a small portion of the surface. Then he waggled his finger as if to count something, drew a couple of figures and a couple of multiplications and subtractions in pencil, and drew out the area in square metres with astonishing precision. He burst out laughing when he saw that the result was right, and added something that I didn't understand much, due to his dialect unknown to me, but he seemed quite happy and satisfied. I was unable to make him explain to me the series of operations he had carried out to arrive at the area of that irregular polygon; however, I am quite sure that Antonio, although he knew the basic mathematical operations, was not very familiar with the basic geometric formulae one learns at school. Yet he had his own way of arriving at the result, which worked, and also seemed to give him a certain satisfaction, like any good problem solving.

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In all likelihood, Antonio's method for calculating area will not end up in teaching manuals; nevertheless, it was effective and, in its self-production, it constitutes a genuine case of what Max Wertheimer calls *productive thinking*: the creative activity of reasoning aimed at identifying solution tools. Within this theoretical framework, in the terms outlined by *Gestalttheorie* of which Wertheimer was one of the founders, the perceptual dimension holds a prominent role (see Wertheimer 1924; Katz 1992). Forms, in fact, are primarily perceptual: for this psychological school, and not secondarily a philosophical one, the whole is the original mode of our sensory interaction with the world. Therefore, if we want to make use of *Gestalt* categories, it will be essential to consider the foundational passage from αἰσθητικός, i.e. that which concerns sensation (in turn from αἰσθάνομαι as 'to feel', 'to perceive'); thus, from the first and most cognitive dimension of aesthetics, which we have referred to from the title as aisthesis. This paper aims to investigate in a first part the connection between mathematical thinking and the perceptual aspect of experience within the framework of *Gestalttheorie*, and in a second part the role of language in the advancement of knowledge and problem solving in this field. For this second part we will use, in continuity with Wertheimer's work, some phenomenological arguments developed in Maurice Merleau-Ponty's work on the connection between language and thought.

1. Blind attitude versus understanding

In mathematical practice, there are at least two ways of working, which are certainly interconnected, but at the same time exercised with very different attitudes by the subject. On the one hand, we have 'calculation' activities: these are operations, usually carried out according to an established sequence in algorithmic form, aimed at processing data in order to obtain output results; the repetitive, serial and automatic dimension is prevalent here, in the context of the application of rules already known to the subject who is operating. In school education, this type of competence is usually learnt through 'exercises', for the performance of which the formula to be applied is indicated upstream, more or less explicitly. For example, if I am assigned a set of binomials to square and I am first taught the formula, I will practice some variations on the same theme by applying a rule to cases for which I know it is relevant.

Another way of practicing mathematics is to deal with 'problems': this occurs when we are confronted with a situation that can be translated into a question requiring an answer. In such cases, we usually do not know in advance exactly what solving protocol is to be adopted; we may have to choose between the mathematical tools we have already acquired, or we may not yet know the appropriate formulae for the solution, or, as is the case in academic research, a solving protocol may not yet have been established for this type of problem.

Wertheimer notes that problem-solving is analogous in its structure of thought, both in cases faced by professional mathematicians and in everyday life situations where the use of mathematics is required. The «birth of a genuine idea, of a productive development» is as much a «surprising event» (Wertheimer 1959, p. 36) for the individual who gives form to his personal method (like our Antonio who we can imagine in front of his first irregular floor plan) as it is for the scientist. It is «productive thinking» (see also Wertheimer 1925):

> What occurs when, now and then, thinking really works productively? What happens when, now and then, thinking forges ahead? What is really going on in such a process? […] when one has just had a creative idea, however modest the issue, when one has begun really to grasp an issue […] There are fine cases. You can find them often, even in daily life […] the transition from a blind attitude to understanding in a productive process (Wertheimer 1959, p. 36).

This contribution by the Czech psychologist constitutes the main study in which the operational concept of *Gestalt* is extended from the perceptual sphere, its main area of relevance, to that of reasoning.[1]

Wertheimer, who was passionate about mathematics and logic[2] and strongly interested in the didactic aspects of these disciplines in the education system, has built his study on a large and solid stock of data, obtained in experimental situations with various groups of subjects.[3] Alongside his own investigations, the author proposes an analysis in *Gestalt* terms of a famous anecdote in which Carl Friedrich Gauss was protagonist as a child.[4] It is said that the task given to him was a long sum of natural num-

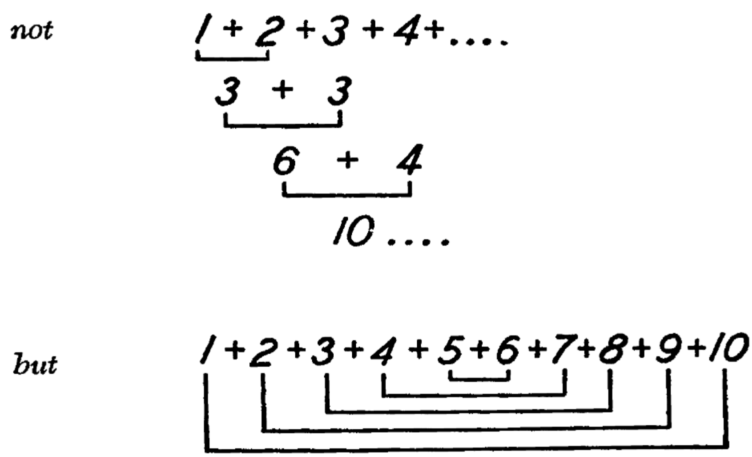
[1] More interesting thoughts on this subject are in Köhler 1969.

[2] On this subject, the testimonies contained in the correspondence with Albert Einstein, his dear friend, are both instructive and fascinating. He often sent Einstein riddles and curious problems enriched with drawings and diagrams: see Luchins, Luchins 1990 and also Miller 1975.

[3] On the issue of the method between quantitative and qualitative approaches, see Paolo Bozzi's very clear introduction to the Italian edition: Bozzi 1997; but also Nerney 1979 for a more critical approach.

[4] The author places the age of the young mathematician at 6 years old and frames the task as a summation from 1 to 10. In fact, the story has been recounted over the years with some variations, including the age and the terms of the summation, which is more often reconstructed as from 1 to 100 (a more plausible scenario, as summing progressively up to 10 would not have constituted a task so lengthy as to require abbreviation). Historical inaccuracies of this kind, however, would have no impact on Wertheimer's reflections regarding the mathematical operation underlying the summation. For an investigation into the reconstruction of the anecdote, see Hayes 2006.

bers in progression; let us imagine, with Wertheimer, that this operation was required on the series from 1 to 10. One could proceed by adding the numbers from first to last, in twos and twos, but Gauss devised a process that avoided this repetitive operation and replaced it with another, faster one. The first step consists of noting that the position of the numbers in the series allows one to associate pairs (first and last number, second and penultimate, and so on with internal pairs), obtaining for each sum the same result, which in our example is 11:[5]



At this point, the number of pairs is needed, which will be equal to half of the last number; multiplying 11 x 5 will give the required result. From this ingenious procedure will derive, later on, Gauss's formula for the sum of the first *n* natural numbers, which states that the sum of the natural numbers from 1 to *n* is equal to the half-product between *n* and $n + 1 : S_n = (n + 1)\frac{n}{2}$.

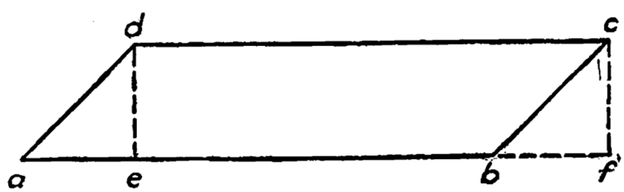
The possibility of reorganising the series into pairs in this specific way does not lie in the individual numbers as such, but in the internal logic of the structure in which they are placed: it is necessary to consider their position value in this specific *whole*. In fact, the succession of natural numbers gives rise to a pattern in which the factors increase (always +1) when viewed from left to right, and consequently decrease in the same way when viewed from right to left. On this progressive trend, of 1 unit ascending or descending at a time, is founded the property of the series whereby the sum of the 'internal' pairs, first-last and so on up to the 'center', must be equivalent; hence, it is on this network of relations between

[5] The fig. 60, so named by the author, is in Wertheimer 1959, p. 125.

the elements that rests the possibility of deconstructing the series and reconstructing it according to new relations. The latter must constitute a sort of *translation* of the first situation into a second, which maintains its initial requirements: attempting random or inappropriate reorganisations would lead to a failure to solve the problem.

Without wishing to take anything away from Gauss's genius, which emerges far beyond this single episode from his childhood, we can draw some interesting considerations from what follows in Wertheimer's investigation: we could have more 'young Gausses', if only we encouraged subjects, during the years of early education, to look for solutions to problems on their own by handing over fewer ready-made formulae. In fact, the various experimental contexts in which the psychologist has confronted young students with mathematical questions similar to the one set out so far, show, along with less functional approaches on the part of some, also several instances of particularly creative, ingenious and effective reasoning processes.

Another example, very simple and probably familiar to many since it is usually encountered in the early years of school, is the geometric procedure by which the equivalence between a parallelogram and a rectangle having congruent bases and heights is demonstrated. This construction is often shown to students to justify using the same formula to calculate the area of the two polygons.[6]



By drawing the segment *de* we isolate, in the starting figure *adcb*, a triangular portion; then projecting the vertex *c* onto the prolongation of the base *ab* we can see a triangular area congruent to that in *ade*, where we can translate the portion of the figure obtained to 'transform' it into a rectangle. The rectangle *edcf*, resulting from the decomposition and reconstruction of the parallelogram *adcb*, constitutes an appropriate reorganisation of it: the problem concerned the calculation of the area, and we note that the area was neither reduced nor extended by the reorganisation, but only reconfigured in such a way as to make its calculation

[6] The fig. 2, so named by the author, is in Wertheimer 1959, p. 50.

easier. We note, incidentally, that usually in primary education one is first taught to calculate the area of squares and rectangles, and one arrives at the parallelogram – and later trapezoids – already knowing the formula. The latter in turn is usually demonstrated to children by dividing the rectangle into squares, adding these up and from there moving on to multiplication of the lengths of the two sides.

Wertheimer also reports here various tasks that he assigns to students, for the calculation of areas of irregular polygons or even figures with a closed curve perimeter, in order to observe the solving processes, set in motion in the absence of formulae to be applied that alone suffice for the result. Some subjects creatively exploit the principle of reorganisation; others went astray or end up overcomplicating the required problem; still others, finally, fail to advance in the request at all, merely staring at the figures or complaining that no one taught them how to solve the problem. The blind attitude, i.e., the habit of repetitively applying already existing protocols without genuine understanding or even giving up in the absence of instructions, is thus opposed to productive thinking, which instead tends towards real mathematical understanding.

2. Functional fixedness

Mathematical problem solving, in *Gestalt* terms, goes through a number of recurring steps that are indispensable to an effective search for the solution. The problem situation must be observed and understood by looking at the network of relationships that link the individual elements together and which, as posited in the foundations of the *Gestalttheorie*, constitute a *whole* whose properties will be different from those of the individual elements merely added together. The solution process requires an appropriate reconfiguration of the relationships between the elements, which identifies «the centering required by the objective structure of the situation» (Wertheimer 1959, p. 169). For example, in the above-mentioned sum, the main 'pattern' that emerges at first glance from observing the series is the arithmetic progression that perceptually 'advances' from left to right. The new structure, leading to the solution, reorganises the progression (visually translatable into a line, or rather a kind of oriented vector) into a pattern symmetrically organised around a center. Furthermore, the process requires «a change of meaning of the parts», i.e. of the function of each piece of the whole, aimed at the transformation of the problem «in terms of "good structure"» (Wertheimer 1959, p. 169), according to the Law of Prägnanz (see Katz 1992). The result will be a «change in structural view» that will have to be thought out «according to the requirements of the problem» (Wertheimer 1959,

p. 160) and that therefore does not transform the starting set into something totally different, but reorganises it while leaving its basic identity preserved. In this same sense, we can say that $S_n = (n+1)\frac{n}{2}$ is another way of indicating 1+2+3+4+5+6+7+8+9+10 = x; a way organised to become a useful tool for solving future problems of the same type.

Whether we are talking about major challenges for the entire scientific community, or minor everyday problems that affect the individual, the solution almost never springs to mind in a flash and without effort. As Wertheimer reminds us, «before the thought process takes place, or in its early stages, one often has a certain whole-view of the situation, and so of its parts, which is somehow unsuited to the problem, superficial, or onesided»; progressing towards the solution requires a certain degree of detachment or elasticity from the first impression. Indeed, «if one sticks to this view, one will often be unable to solve the actual problem» (Wertheimer 1959, p. 160).

Karl Duncker, who worked with Wertheimer, dealt specifically with difficulties in mathematical problem solving and proposed a hypothesis about the main hindering factors for *Gestalt* restructuring. By examining the reasons why some subjects can define themselves as a «"good" mathematician» while others turn out to be «"poor" mathematicians» (Duncker 1945, p. 110), he highlights the role of a condition he calls «functional fixedness» (Duncker 1945, p. 85). This difficulty consists in a reduced or poor capacity for abstraction from the perceptual datum, whereby the sufferer would be able, little or not at all, to derive the underlying pattern of relations from a starting situation. The function of the individual elements remains fixed, rigidly anchored to the specific case; the subject feels it as immutable, and this prevents him from thinking that the elements can undergo a role switch.

Wolfgang Köhler's experiments with apes show (see Köhler 1925), outside of mathematics, this type of difficulty very clearly. These animals, if provided with a wooden branch, are usually able to see the branch as a tool and use it to move any objects in the vicinity, e.g. to bring food closer. On the other hand, as Duncker notes (Duncker 1945, p. 85), if the branch is still attached to the tree – let us assume it is thin enough and low enough to be broken off – they cannot recognise it as an isolable element that can serve as a stick. The structure 'tree' in this case is perceived rigidly, its constituent elements fail to be grasped as such in view of a functional reorganisation of the problem. Difficulties in mathematics are read in the same terms: the starting situation is perceived rigidly, functions are seen as fixed and therefore cannot be moved. On these bases, we can also say that, in order to authentically understand that is the area of the square, I have to go from the perception of a drawn square and make, for example, that those 10 cm drawn with chalk on the blackboard become, a symbol for *side*, as well as realise that behind that number 2 in the superscript is the multiplication.

It follows from what has been said that the aesthetic perceptual aspect is indispensable for accessing the *Gestalt* of the situation as a relational structure between elements, but it also follows that this level must not be overpowering. Indeed, the main difficulties in the mathematical attitude would arise from the excessive adherence to the singular situation in terms of the perceived datum, which remains «alive», «relatively inelastic, rigid, and therefore not sufficiently plastic to be reshaped» (Duncker 1945, p. 108, 110).

There is also an interesting aspect, perhaps worthy of more emphasis than in Duncker's text. Describing the experimental situation that the author sets up to investigate solving processes, he reports suggesting that the subjects help themselves by speaking aloud or drawing: «after the subject was given paper and pencil, I gave […] general instructions to him at the outset of the experimental series: […] *Try to think aloud*. I guess you often do so when you are alone and working on a problem. And draw as much as possible» (Duncker 1926, p. 664). We highlight the following annotation, placed in brackets in the text: «thinking aloud is not an introspective report, no more than drawing would be» (Duncker 1926, p. 664). There is no particular explanation for this suggestion; the author seems to take it for granted, without justifying it. It is not difficult to understand its meaning; one may venture that the use of pen and paper, soliloquy or verbalisation to an interlocutor are familiar means to anyone attempting to solve a problem. However, we would like to attempt a more solid explanation, which will have to pass through the role of language in its various forms involved. In the case of mathematical problem solving, this is first and foremost formal algebraic language; then, usually, support in the historical-natural language in which the subject habitually expresses himself; finally also drawing, which can be considered in the guise of an iconic sign.[7]

3. The mathematical phenomenon

It will be useful, at this point, to consider the reflection on mathematical thought developed by Merleau-Ponty, in the context of his perceptual phenomenology that owes so much to *Gestalt theory*. Recalling the theorem of the sum of the internal angles of a triangle, which establishes that

[7] We suggest, in this regard, a broader comparison with Charles Peirce's semiotic theory, following the insights proposed by Sini 2019, p. 19. Additionally, see the comparison drawn between pictorial signs and language in Merleau-Ponty 1952.

this sum is equivalent to a flat angle (180°), let us recall that the demonstration in turn rests on the theorem of parallel lines cut by a transversal, which establishes precise relationships between internal and external angles. The initial question requires a construction, namely that we can evaluate the starting structure (the triangle) and understand if and how we can modify the starting design in order to obtain a more informative result (Merleau-Ponty 1945, pp. 405-406).

Such an operation is possible thanks to certain characteristics typical of human intentionality, in the phenomenological sense, which rests on the sensory system without being reduced to it. The demonstration in question is due to the fact that «my perception of the triangle was not, so to speak, congealed and dead; the drawing of the triangle on the paper was merely its envelope, it was shot through by lines of force, untraced yet possible directions were born everywhere in it», through which the triangle «was bursting with indefinite possibilities of which the construction actually drawn is merely one particular case» (Merleau-Ponty 1945, p. 406). This reflection, while belonging to a philosophical system that is purely phenomenological, is essentially compatible with Duncker's reflection on functional fixedness: for those afflicted by it, these «indefinite possibilities» are precisely that which does not appear, and the triangle will remain just an immovable triangle. For Merleau-Ponty, therefore, geometric construction results as the product of «the ability I have of making sensible emblems appear from a certain hold on things that is nothing other than my perception of the structure "triangle"» (Merleau-Ponty 1945, p. 406).

In this, as in every other aspect of the phenomenology of perception, it is crucial to recognise the anchoring of knowledge to the human condition of being corporeal. Not only must it be remembered that the subject has a body, but also that cannot not have a body; it is the «body schema» (see Merleau-Ponty 1945, pp. 100-105) that makes each of us «a point of view upon things», and there is no reason why this condition should not also hold true in the field of mathematical thought. In this view, every possible *noesis* is filtered through the support of the body, giving rise to an interaction with the world that is always also spatial; likewise, «the subject of geometry is a motor subject» (Merleau-Ponty 1945, p. 314, 406; see also Halák 2022). Therefore, in the technical lexicon of this discipline, «if the words "on", "by", etc., maintain a sense, this is because I am working on a sensible or imaginary triangle, which is to say, one that is at least virtually situated in my perceptual field, oriented according to "up" and "down", "right" and "left"», i.e. on a figure that is in any case always «implicated in my general hold upon the world» (Merleau-

Ponty 1945, p. 405). One usually remembers that the triangle, the circumference or the straight line as geometric entities are not equivalent to the triangles, circles and lines that we find or reproduce in the physical world, since in their definitions specific properties have been postulated that can never or rarely be satisfied by material entities. However, without invalidating these postulates, we cannot avoid a periodic return to the world of things in the mathematical realm as well: it is the latter that ultimately founds our possibility of understanding and reasoning. Merleau-Ponty emphasises that «there would be no experience of truth» if we reasoned only «*vi formae*, and if formal relations were not first presented to us as crystallized in some particular thing», and that «we would not even be capable of fixing an hypothesis in order to draw out its consequences if we did not begin by taking it as true» (Merleau-Ponty 1945, p. 405). This reflection is certainly not limited to geometry; by means of examples relating to numerical series, such as that of Gauss, which Merleau-Ponty also takes up, it is emphasised that transformations of structure are to be considered «the equivalent in the arithmetical object of a geometric construction» (Merleau-Ponty 1969, pp. 125-126).

There is another aspect to consider: the possibility of «formalization» (Merleau-Ponty 1945, p. 405). In order for a productive thinking process to contribute to the advancement of collective mathematical knowledge, for the scientific community and not just for the individual, it is essential that this process be translated into a language, which cannot be the natural language but must be the one typical of mathematics. «The algorithmic expression», this is the name Merleau-Ponty gives to the formal algebraic and logical language, «is *exact* because of the exact equivalence established between its initial relations and those that are derived from them» (Merleau-Ponty 1969, p. 128). One could compare formal language with natural language on the basis of the degree of semantic vagueness; this approach would lead us to separate the two types of language particularly sharply. Merleau-Ponty points out, in this regard, that algorithmic expression is still «a special case of language», and specifically that it is «secondary». In formal language, the signifier seems considerably closer to the signified than in everyday language; we are led to believe that, unlike historical-natural language, in this symbolic system « the signification is captured without remainder». According to the phenomenologist, this would be an effect of «the *shift* of restructuring», a process that is a «characteristic of language» attributable to the erasure of the transformations undergone by the structure in question. When we have found the formula, we collectively *forget* the steps that led us to it from the problem, with a systematically «omitted mention of the structure's transcendence toward its transformations» (Merleau-Ponty 1969, p. 128).[8]

The centrality of perception in Merleau-Ponty's phenomenological theory does not in any way imply a reduction of thought to it; it is as obvious as it is important to remember that «the thing thought is not the thing perceived», that «knowledge is not perception» and, although comparable to gesture, «is not one gesture among all the other gestures», as it «is the vehicle of our movement toward truth» (Merleau-Ponty 1969, p. 129; see also 1951). Merleau-Ponty argues in depth that there is no pre-existing thought to language: they take shape together, according to a movement that is not to be understood as parallel but as a kind of reciprocal and incessant sculpting of one upon the other. If thought were given first and then speech were to follow, we could not even understand «why the most familiar object appears indeterminate so long as we have not remembered its name» (Merleau-Ponty 1945, p. 182). From this, one can reflect on the function of language, which we could define as heuristic: it is indispensable for the subject not only to communicate with others, but even more so to understand for himself what he intends to communicate. Thought needs a 'body' to take shape and manifest, and conversely, the sign is in turn shaped by thought: «all truly expressive speech» and «all language in the phase in which it is being established» (not just verbal language, but also drawing or painting, and even formal language) does not merely «choose a sign for an already defined signification, as one goes to look for a hammer in order to drive a nail» (Merleau-Ponty 1952, p. 46), but makes the semiotic system act upon meaning to know it in turn. «The thinking subject himself is in a sort of ignorance of his thoughts so long as he has not formulated them for himself, or even spoken or written them» (Merleau-Ponty 1945, p. 183), and there is no reason why this should not also be valid for mathematical thought.

Now, we can return to Duncker's suggestion for facilitating productive thinking: why would speaking aloud or drawing help in solving mathematical problems? Why would it constitute an important step in carrying out reasoning and restructuring the problem? As Merleau-Ponty argues, «for the speaking subject, to express is to become aware of; he does not express just for others, but also to know himself what he intends» (Merleau-Ponty 1951, p. 90): I must move from attributing a 'body' to thought in order to give it shape, and this body con-

[8] It is not an avoidable process, nor is the intention to highlight a problem that needs correction. Rather, it is a characteristic of every language, particularly pronounced in formal ones, which we can assume to be linked to the possibility of syntactic manipulation typical of inference rules in symbolic logic, for example.

sists in the materiality of the sign, whether formal, natural, or iconic. Not only that: I could carry out this operation mentally as well; so why 'aloud' or on paper? We can hypothesize that the dimension of the 'return' of the message from the outside helps facilitate the dynamics of understanding. 'Aloud,' I can listen to myself as if someone else were speaking to me; by drawing, I can look at the paper as if someone else had drawn it. These modes seem to facilitate and strengthen my dual role as both the sender and recipient of my own message, a role that also takes place – we might say with more difficulty and less clarity – in tacit reasoning and without the support of writing. As Merleau-Ponty notes, «to the extent that what I say has meaning, I am a different "other" for myself when I am speaking; and to the extent that I understand, I no longer know who is speaking and who is listening» (Merleau-Ponty 1951, p. 97).

This journey has aimed to show that the *Gestalt theory* applied to thinking, combined with the integration of the Merleau-Pontian phenomenological theory, allows for an exceptionally fruitful exploration of the relationship between mathematics, language, and the aesthetic dimension. The latter has been worked through and understood in this contribution in its broadest sense as knowledge tied to sensoriality; we might add in passing that this reflection could also be seen as a useful element for understanding the phenomenon of the relevance of beauty in mathematics. Indeed, a form particularly suited to conveying a structure of meaning (which can also be constituted by a formal linguistic expression), being its *material* aspect, could be considered a perceptible, aesthetic object, capable of being appreciated for its beauty as well.[9] Setting aside this additional point, let us return to the brief quotation at the beginning, in which Duncker emphasizes how authentic understanding is linked to the possibility of grasping a situation in its *Gestalt*, that is, in its network of foundational relations, beyond the mere sum of its constituent elements. We find a striking correspondence in Merleau-Ponty's consideration, which sheds further light on the application of phenomenology to the mathematical field: «what is essential», — and here we refer to the specific phenomenological sense of essence — «to mathematical thought, therefore, lies in the moment where a structure is decentered, opens up to questioning, and reorganizes itself according to a new meaning» showing that « the truth of the result», ultimately, «lies in a restructuring» (Merleau-Ponty 1945, pp. 160, 404).

[9] We have discussed this hypothesis in detail in De Rosa 2023.

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Note per un'*aisthesis* matematica: dalla *Gestalt* al linguaggio

Questo articolo esplora alcune possibilità di indagine del pensiero matematico in chiave percettiva, in particolar modo concentrandosi sul possibile nesso tra i processi creativi matematici e le forme (*Gestalten*) rilevabili durante tali processi. A tale scopo, una prima parte è dedicata alla prospettiva fornita da autori nel quadro della *Gestalttheorie*, sia in campo matematico che nella più ampia indagine sui processi di soluzione dei problemi; una seconda parte, avvalendosi della continuità tra ricerche gestaltiste e metodo fenomenologico, è dedicata alla riflessione sulla funzione del linguaggio nell'avanzamento della conoscenza in ambito matematico.

PAROLE CHIAVE: Teoria della forma, Filosofia della matematica, Fenomenologia, Wertheimer, Merleau-Ponty

Notes for a mathematical *aisthesis*: from *Gestalt* to language

This article explores some possible ways of investigating mathematical thinking from a perceptual perspective, focusing in particular on the potential link between creative mathematical processes and the forms (*Gestalten*) that can be grasped during such processes. To this end, the first part is devoted to the perspective offered by authors within the framework of *Gestalt theory*, both in the mathematical field and in the broader investigation of problem-solving processes; the second part, drawing on the continuity between *Gestalt* research and the phenomenological method, is dedicated to reflecting on the role of language in the advancement of knowledge in the field of mathematics.

KEYWORDS: Gestalt theory, Philosophy of mathematics, Phenomenology, Wertheimer, Merleau-Ponty